

Binary Arithmetic

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August 2010

Remember how the decimal system works

Numbers are organized into columns. 983 is:

Hundreds	Tens	Ones
9	8	3

This is in powers of 10

10^2	10^1	10^0
9	8	3

Binary numbers work the same way. 6 is:

Fours	Twos	Ones
1	1	0

2^2	2^1	2^0
1	1	0

Arithmetic works the same way

$$7 + 5 = 12$$

	2^3 8	2^2 4	2^1 2	2^0 1
7 =	0	1	1	1
5 =	0	1	0	1
12 =	1	1	0	0

Negative numbers are trickier

Computers do not have plus and minus signs!

- One solution is to use a fixed number of bits for arithmetic and to use the left-most bit as the sign bit. Suppose we have 8 bits, $12 = 00001100$ and $-12 = 10001100$
- Then the number range is $\pm 2^7 - 1 = \pm 127$
- One's Complement: flip all the 0s and 1s; keep sign bit
 $-12 = 11110011$
- Two's Complement: Start with 1's complement and add 1 if it is negative:
 $11110011 + 1 = 11110100$
- Excess $2^{(m-1)}$ [$m=7$ for us]: Add 2^7 to each number
 $7 = 128 + 7 = 135 = 10000111$; $-7 = 128 - 7 = 121 = 01111001$
Note that's this is the same as the twos complement with the left bit flipped.

You need to know which representation was used!

Examples

Using the regular algorithm for binary addition, add (5+12), (-5+12), (-12+-5), and (12+-12) in each system. Then convert back to decimal numbers.

Signed Magnitude:

5+12	-5+12	-12+-5	12+-12
00000101 00001100	10000101 00001100	10001100 10000101	00001100 10001100
<u>00010001</u>	<u>10010001</u>	<u>00010001</u>	<u>10011000</u>
17	-17	17	-24

One's Complement:

5+12	-5+12	-12+-5	12+-12
00000101 00001100	11111010 00001100	11110011 11111010	00001100 11110011
<u>00010001</u>	<u>00000110</u>	<u>11101101</u>	<u>11111111</u>
17	6	-18	0

Two's Complement:

5+12	-5+12	-12+-5	12+-12
00000101 00001100	11111011 00001100	11110100 11111011	00001100 11110100
<u>00010001</u>	<u>00000111</u>	<u>11101111</u>	<u>00000000</u>
17	7	-17	0

Excess 2^m:

5+12	-5+12	-12+-5	12+-12
10000101 10001100	01111011 10001100	01110100 01111011	00001100 01110100
<u>00010001</u>	<u>00000111</u>	<u>11101111</u>	<u>01111100</u>
109	119	111	124

Two's complement works!

How to do two's complement fast

Flip all the bits including the sign bit and add one

00001100 = 12;

11110011

+ 1

—————

11110100 = -12 in two's complement

It is critically important to remember that the place of the negative-weight bit must be already determined before any two's complement conversions can be done.